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CS 471
Homework 5
03/23/19

1. Give a joint distribution for Boolean random variables A, B, and C for each scenario. Give a brief intuitive interpretation of the variables. The notation i(x, y) means that x and y are independent.
   a. i(A, B), i(A, C), and i(B, C)

      P(A, B, C) = **P**(A)**P**(B)**P**(C)

      All variables, A, B, and C are independent from each other; there are no dependent variables in this scenario.

   b. i(A, B) and i(A, C), but not i(B, C)

      P(A, B, C) = **P**(A)**P**(B | C)**P**(C)

      B and C are dependent (or at least not proven to be independent). All other pairings are independent.

   c. i(A, B) and i(A, C), but not i(A, B $\wedge$ C)

      P(A, B, C) = **P**(A | B, C)**P**(B)**P**(C) [Assuming B and C are independent]

      A is independent of the portions of B and C which do not overlap each other.

2. Given the full joint distribution shown in Figure 13.3, calculate the following:
   a. P(toothache)

      = 0.108 + 0.012 + 0.016 + 0.064 = 0.2

   b. **P**(Cavity)

      = $\langle$ 0.108 + 0.012 + 0.072 + 0.008, 0.016 + 0.064 + 0.144 + 0.576 $\rangle$
      = $\langle$ 0.2, 0.8 $\rangle$

   c. **P**(Toothache | cavity)

      = $\langle$ (0.108 + 0.012) / 0.2, (0.072 + 0.008) / 0.2 $\rangle$
      = $\langle$ 0.6, 0.4 $\rangle$

   d. **P**(Cavity | toothache $\vee$ catch)

      = $\langle$ (0.108 + 0.012 + 0.072) / (0.108 + 0.012 + 0.016 + 0.064 + (0.072 + 0.144)), (0.016 + 0.064 + 0.144) / (0.108 + 0.012 + 0.016 + 0.064 + (0.072 + 0.144)) $\rangle$
      = $\langle$ 0.461538, 0.538461 $\rangle$

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3. Suppose you are a witness to a nighttime hit-and-run accident involving a taxi in Athens. All taxis in Athens are blue or green. You swear, under oath, that the taxi was blue. Extensive testing shows that, under the dim lighting conditions, discrimination between blue and green is 70% reliable.

   a. Is it possible to calculate the most likely color for the taxi? (Hint: distinguish carefully between the proposition that the taxi is blue and the proposition that it appears blue.)

      Let B be the event the taxi was blue, and A be the event the taxi appeared blue. Then from the question we know that,
      $P(A|B) = 0.7$ and
      $P(\neg A|\neg B) = 0.7$

      We would like to know the probability the taxi was actually blue given that it appeared blue, which is modeled by $P(B|A) \propto P(A|B)P(B) \propto 0.7\,P(B)$. And for the opposing case where the taxi was not actually blue given that it appeared blue, $P(\neg B|A) \propto P(A|\neg B)P(\neg B) \propto (1-0.7)(1-P(B))$.

      So, if we are given a tangible probability for a taxi being blue, this is solvable. Otherwise, we can just talk about it generally without having a concrete answer.

   b. What if you know that 8 out of 10 Athenian taxis are green?

      If 8 in 10 taxis are green, then P(B) = 0.2. We may then calculate the most likely color for the taxi.

      $$P(B|A) \propto 0.7(0.2) \propto 0.14 \Rightarrow P(B|A) = \frac{0.14}{0.14+0.56} = 0.2$$
      $$P(\neg B|A) \propto 0.7(0.8) \propto 0.56 \Rightarrow P(\neg B|A) = \frac{0.56}{0.14+0.56} = 0.8$$

4. We have a bag of three biased coins a, b, and c with probabilities of coming up heads of 40%, 40%, and 80%, respectively. One coin is drawn randomly from the bag (with equal likelihood of drawing each of the three coins), and then the coin is flipped three times to generate the outcomes X_1, X_2, and X_3.

   a. Draw the Bayesian network corresponding to this setup and define the necessary CPTs.

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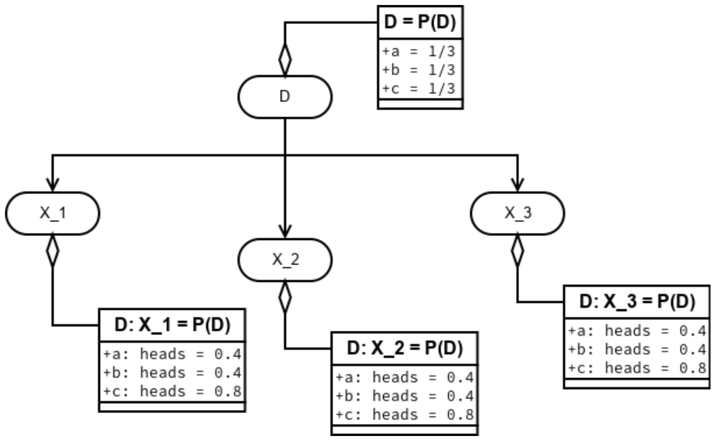

Where D is the event representing the drawing of a coin.

b. Calculate which coin was most likely to have been drawn from the bag if the observed flips come out heads twice and tails once.

We are aiming to maximize the probability of P(D | 1 tail, 2 head) by changing the coin drawn.

P(D | 1 tail, 2 head)

$$=\frac{P(1 \text{ tail}, 2 \text{ head} \mid D)P(D)}{P(1 \text{ tail}, 2 \text{ head})}$$

$\propto P(1 \text{ tail}, 2 \text{ head} \mid D)$ because P(D) and P(1 tail, 2 head) are both independent of D.

In the case of coin a, the conditional probability is as follows:

$P(X_1 = tails, X_2 = heads, X_3 = heads \mid D = a)$ and since the X_i's are independent given the drawn coin,

$P(X_1 = tails \mid D = a) P(X_2 = heads \mid D = a) P(X_3 = heads \mid D = a)$

$= 0.6 * 0.4 * 0.4 = 0.096$

And since there are 3 possible ways to arrange the coin tosses,

$P(1 \text{ tail}, 2 \text{ head} \mid D = a) = 3 * 0.096 = 0.288$

Following similar logic for coins b and c,

$P(1 \text{ tail}, 2 \text{ head} \mid D = b) = 3 * 0.096 = 0.288$

$P(1 \text{ tail}, 2 \text{ head} \mid D = c) = 3 * 0.128 = 0.384$

Therefore, coin c is the most likely coin to have been drawn.

5. In your local nuclear power station, there is an alarm that senses when a temperature gauge exceeds a given threshold. The gauge measures the temperature of the core.

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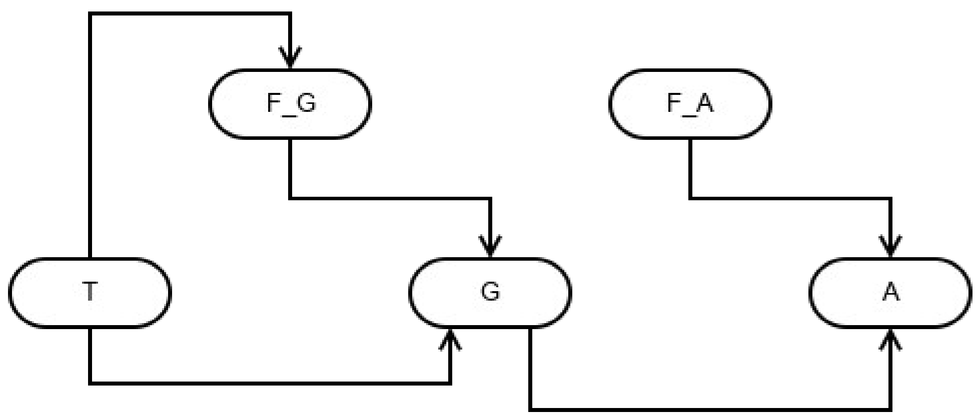
Consider the Boolean variables A (alarm sounds), F_A (alarm is faulty), and F_G (gauge is faulty) and the multivalued nodes G (gauge reading) and T (actual core temperature).

a. Draw a Bayesian network for this domain, given that the gauge is more likely to fail when the core temperature gets too high.



b. Is your network a polytree? Why or why not?

This is not a polytree because the temperature (T) effects both the gauge (G) and it's failure (F_G).

c. Suppose there are just two possible actual and measured temperatures, normal and high; the probability that the gauge gives the correct temperature is x when it is working, but y when it is faulty. Give the conditional probability table associated with G.

	T = normal		T = high	
	F_G	$\neg F_G$	F_G	$\neg F_G$
G = normal	y	x	1 - y	1 - x
G = high	1 - y	1 - x	y	x

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d. Suppose the alarm works correctly unless it is faulty, in which case it never sounds. Give the conditional probability table associated with A.

	G = normal		G = high	
	F_A	$\neg F_A$	F_A	$\neg F_A$
A	0	0	0	1
$\neg A$	1	1	1	0

e. Suppose the alarm and gauge are working and the alarm sounds. Calculate an expression for the probability that the temperature of the core is too high, in terms of the various conditional probabilities in the network.

The question is modeled by $P(T\,|\,A,\neg F_G,\neg F_A)$, where T and G as represented from here on are high temperature. This can be reduced to $P(T\,|\,A,\neg F_G,G)$ when we realize that T is d-separated from F_A and A.

$$P(T\,|\,A,\neg F_G,G)\propto P(G|\,T,\neg F_G)P(T|\neg F_G)$$
$$P(G|\,T,\neg F_G)P(T\,|\neg F_G)\propto P(G|\,T,\neg F_G)P(\neg F_G|\,T)P(T)$$

And similarly for $\neg T$,

$$P(\neg T\,|\neg F_G,G)\propto P(G|\neg T,\neg F_G)P(\neg F_G|\neg T)P(\neg T)$$

Then we normalize,

$$P(T\,|\neg F_G,G)=\frac{P(G|\,T,\neg F_G)P(\neg F_G|\,T)P(T)}{P(G|\,T,\neg F_G)P(\neg F_G|\,T)P(T)+P(G|\neg T,\neg F_G)P(\neg F_G|\neg T)P(\neg T)}$$

If we insert the probabilities x and y from above, and letting $t=P(T)$, $f=P(F_G|\,T)$, and $d=P(F_G|\neg T)$, then

$$P(T\,|\neg F_G,G)=\frac{(t)(1-f)(1-x)}{(t)(1-f)(1-x)+(1-p)(1-d)(x)}$$