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CS 471
Homework 3
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1. Does the fact ¬BestFriend(Mary, Sophia) follow from the facts Mary ≠ Katie and BestFriend(Katie, Sophia)? If so, give a proof; if not, supply additional axioms as needed. What happens if we use BestFriend as a unary function symbol instead of a binary predicate?

   This fact does not follow. We need a fact that indicates that each person may only have a single Best Friend. An additional axiom that may solve this would be:

   $\forall a, b, c \;\; \text{BestFriend}(a,b) \wedge \text{BestFriend}(c,b) \Rightarrow a = c$

   If BestFriend were a unary function, then the fact would follow. This is because $BestFriend(Sophia) = Katie$ and $\neg BestFriend(Sophia) = Mary$ alongside the fact $Mary \neq Katie$ leads to a situation where the unary function BestFriend may only have one output mapped to the input Sophia (which cannot be both Mary and Katie).

2. Assuming predicates Parent(p, q) and Male(p) and constants Roger and Amy, with the obvious meanings, express each of the following sentences in first-order logic. (You may use the abbreviation $\exists^1$ to mean there exists exactly one.)

   a. Roger has a Son (possibly more than one, and possibly daughters as well).

      $\exists c \;\; \text{Parent}(\text{Roger}, c) \wedge \text{Male}(c)$

   b. Roger has exactly one son (but may have daughters as well).

      $\exists^1 c \;\; \text{Parent}(\text{Roger}, c) \wedge \text{Male}(c)$

   c. Roger has exactly one child, a son.

      $\forall c \;\; \text{Parent}(\text{Roger}, c) \wedge Male(c) \wedge (\forall t \;\; \text{Parent}(\text{Roger}, t) \Rightarrow c = t)$

   d. Roger and Amy have exactly one child together.

      $\exists^1 c \;\; \text{Parent}(\text{Roger}, c) \wedge \text{Parent}(\text{Amy}, c)$

   e. Roger has at least one child with Amy, and no children with anyone else.

      $\exists c \;\; \text{Parent}(\text{Roger}, c) \wedge \text{Parent}(\text{Amy}, c) \wedge$
      $(\forall s, t (\text{Parent}(\text{Roger}, s) \wedge \text{Parent}(t, s)) \Rightarrow (t = \text{Roger} \vee t = \text{Amy}))$

3. For each pair of atomic sentences, give the most general unifier if it exists:

   a. Q(A, B, B), Q(x, y, z).

      $\{x/A, y/B, z/B\}$

   b. P(y, R(A, B)), P(R(x, x), y).

      A unifier does not exist for this since x may not be both A and B.

   c. Younger(Mother(y), y), Younger(Mother(x), John).

      $\{x/\text{John}, y/\text{John}\}$

   d. Knows(Son(y), y), Knows(x, x).

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A unifier does not exist because y cannot be unified with Son(y).

4. Here are two sentences in the language of first-order logic:
   (A) ∀x ∃y (x ≥ y)
   (B) ∃y ∀x (x ≥ y)
   a. Assume that the variables range over all the natural numbers 0, 1, 2, . . . , ∞ and that the ”≥” predicate means is greater than or equal to. Under this interpretation, translate (A) and (B) into English.

      A means: “For all natural numbers, there is another natural number which is less than or equal to it.”

      B means: “There is a certain natural number that is less than or equal to all natural numbers.”

   b. Is (A) true under this interpretation?

      A is true because y may be the same number as x due to it being greater than or *equal* to.

   c. Is (B) true under this interpretation?

      B is true because y may be set to 0 which will equal all natural numbers.

   d. Does (A) logically entail (B)?

      No, A does not entail B.

   e. Does (B) logically entail (A)?

      Yes, B entails A.

   f. Using resolution, try to prove that (A) follows from (B). Do this even if you think that (B) does not logically entail (A); continue until the proof breaks down and you cannot proceed (if it does break down). Show the unifying substitution for each resolution step. If the proof fails, explain exactly where, how, and why it breaks down.

      We would like to show $A \vDash B$. Let our knowledge base (KB) contain $A, \neg B$. We must Skolemize the statements as so: $A\colon\ x \geq F(x)$ and $\neg B\colon\ \neg G(y) \geq y$. We find that we may not unify these statements. The proof breaks down when the proper unification would be $\{x/G(y), y/F(x)\}$; however, this then equals $\{x/G(y), y/F(G(y))\}$ which is then invalid because y is recursively defined. Thus, the resolution fails and A does not entail B.

   g. Now try to prove that (B) follows from (A).

      We would like to show $B \vDash A$. Let our knowledge base (KB) contain $\neg A, B$. This leads to the statements: $\neg A\colon\ \neg F \geq y$ and $B\colon\ x \geq G$. The unification succeeds with $\{x/F, y/G\}$. Thus, the resolution succeeds, returns false, and B does entail A.