CS471- Homework 3

Due date: Feb 17th at 11:59pm EST

Introduction

The objective of this assignment is to get familiar with First-Order Logic.

Submission

Your homework must be typed and must contain your name and Purdue ID. To submit your assignment, log into `data.cs.purdue.edu` (physically go to the lab or use ssh remotely) and follow these steps:

1. To ssh use the command: `ssh username@data.cs.purdue.edu`
2. Make a directory named `username-hw3` (all letters in lower case)
3. Copy your PDF and code inside it. To do it remotely use the comand from your computer:
   `scp ./path/to/your-file.pdf username@data.cs.purdue.edu:./remote/path/from-home-dir/`
4. Go to the directory containing `username-hw3` (e.g., if the files are in /homes/aporco/aporco-hw3, go to /homes/aporco), and execute the following command:

   `turnin -c cs471 -p hw3 username-hw3`

   (e.g. Aldo would use: `turnin -c cs471 -p hw3 aporco-hw3` to submit his work)
5. To overwrite an old submission, simply execute this command again.
6. To verify the contents of your submission, execute the following command:

   `turnin -v -c cs471 -p hw3`

Required files

You will need to submit 1 file:

- The PDF containing your typed answers.

Problem Set

Problem 1: Does the fact $\neg BestFriend(Mary, Sophia)$ follow from the facts $Mary \neq Katie$ and $BestFriend(Katie, Sophia)$? If so, give a proof; if not, supply additional axioms as needed. What happens if we use BestFriend as a unary function symbol instead of a binary predicate?

Problem 2 Assuming predicates $Parent(p, q)$ and $Male(p)$ and constants $Roger$ and Amy, with the obvious meanings, express each of the following sentences in first-order logic. (You may use the abbreviation $\exists$ to mean there exists exactly one.)

a. Roger has a Son (possibly more than one, and possibly daughters as well).
b. Roger has exactly one son (but may have daughters as well).
c. Roger has exactly one child, a son.
d. Roger and Amy have exactly one child together.
e. Roger has at least one child with Amy, and no children with anyone else.

Problem 3: For each pair of atomic sentences, give the most general unifier if it exists:

a. $Q(A, B, B)$, $Q(x, y, z)$.
b. $P(y, R(A, B))$, $P(R(x, x), y)$.
c. $Younger(Mother(y), y)$, $Younger(Mother(x), John)$.
d. $Knows(Son(y), y)$, $Knows(x, x)$.

Problem 4: Here are two sentences in the language of first-order logic:

(A) $\forall\ x\ \exists\ y\ (x \geqslant y)$
(B) $\exists\ y\ \forall\ x\ (x \geqslant y)$

a. Assume that the variables range over all the natural numbers $0, 1, 2, \ldots, \infty$ and that the "$\geqslant$" predicate means is greater than or equal to. Under this interpretation, translate (A) and (B) into English.
b. Is (A) true under this interpretation?
c. Is (B) true under this interpretation?
d. Does (A) logically entail (B)?
e. Does (B) logically entail (A)?
f. Using resolution, try to prove that (A) follows from (B). Do this even if you think that (B) does not logically entail (A); continue until the proof breaks down and you cannot proceed (if it does break down). Show the unifying substitution for each resolution step. If the proof fails, explain exactly where, how, and why it breaks down.
g. Now try to prove that (B) follows from (A).